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The shadow rate as a predictor of real activity and inflation: Evidence from a data-rich environment

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Abstract

This paper examines the predictive content of the shadow rates for U.S. real activity and inflation in a data-rich environment. We find that the shadow rates contain substantial out-of-sample predictive power for inflation in non-zero lower bound and zero lower bound periods. In contrast, the shadow rates are uninformative about future real activity.

Keywords: shadow rate, zero lower bound, unconventional monetary policy, forecasting, data-rich environment

JEL codes: C53, E37, E43, E44, E58

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1. Introduction

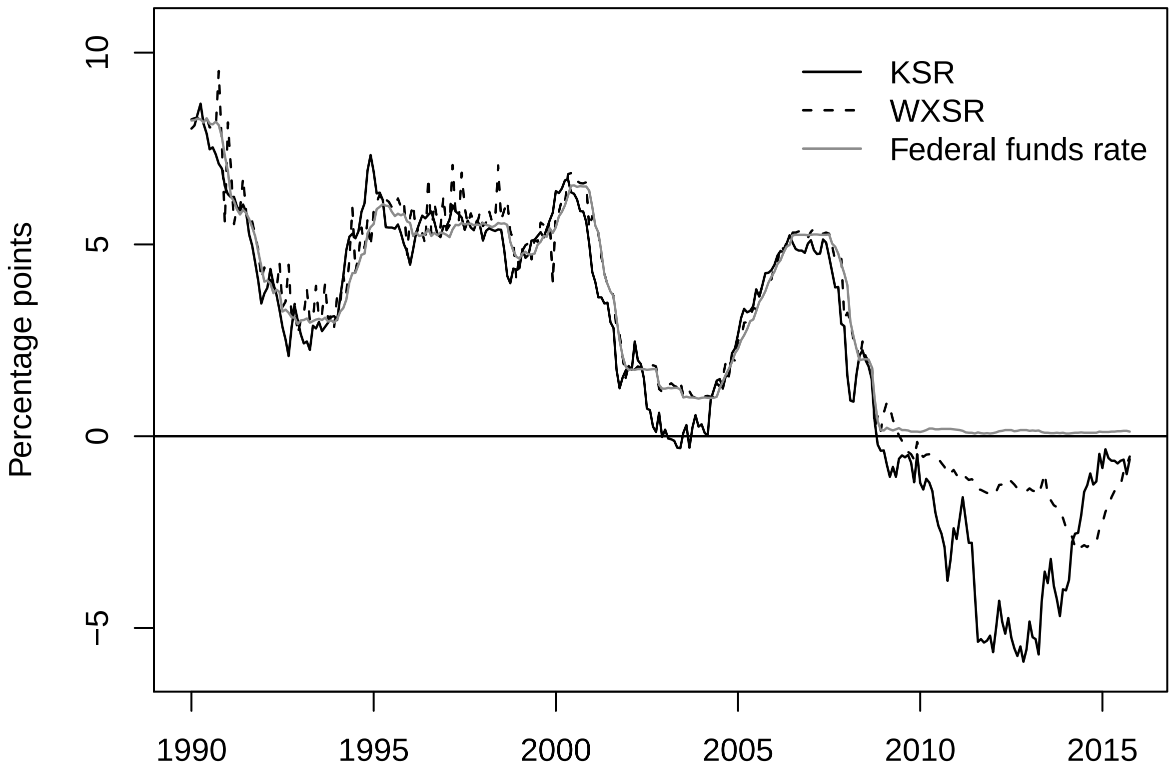
Historically, the federal funds rate (henceforth, FFR) has been the primary instrument of monetary policy. The Fed has lowered the FFR to boost economic activity and raised the rate when the economy is overheating. The empirical literature has shown that short-term rates have predictive power for future economic activity (Ang et al., 2006).

The financial crisis in 2008 changed the Fed's monetary policy altogether. The FFR was stuck at the zero lower bound (ZLB) from December 2008 to December 2015. During this period, the Fed used unconventional monetary policies such as quantitative easing and forward guidance.

When the ZLB is binding, the FFR does not display meaningful variation and thus no longer conveys information about the monetary policy stance. Krippner (2015b) and Wu and Xia (2016) argue that the shadow rate can be used in place of the FFR to describe the stance and effects of the monetary policy in the ZLB environment. When the FFR is stuck at the ZLB, the shadow rate can freely take on negative values to reflect unconventional monetary policy actions. The shadow rate has received a lot of attention in the recent literature (see, e.g., Wu and Xia, 2016 and the references cited therein). However, to our knowledge, the predictive ability of the shadow rate for future economic activity has not been examined. As an indicator of an effective monetary policy, the shadow rate is potentially informative about the future state of the economy.

This paper contributes to the existing literature by analyzing the predictive content of the shadow rates for U.S. real activity and inflation in a data-rich environment. Because Bauer and Rudebusch (2013) find that different model specifications produce different estimates of the shadow rate, we consider two versions of the shadow rate in our forecasting exercise. The first one is suggested by Krippner (2015b; henceforth

Figure 1: Shadow rates and the federal funds rate



Notes: The sample period is 1990:M1–2015:M10.

KSR), and the second is introduced by Wu and Xia (2016; henceforth WXSR). Figure 1 plots the KSR, WXSR and FFR from 1990:M1 to 2015:M10.

The main finding from this study is that the shadow rates contain predictive power for inflation but not for real activity. The WXSR is a more informative leading indicator than the KSR. The WXSR contains substantial predictive power for inflation in the non-ZLB and ZLB periods.

2. Methods

Our forecasting model is the following linear, h-step-ahead dynamic factor model, augmented with a shadow rate:

$$y_{t+h}^{h} = \alpha_h + \sum_{j=1}^{m} \sum_{i=1}^{k} \beta_{hij} \hat{F}_{i,t-j+1} + \sum_{j=1}^{p} \gamma_{hj} y_{t-j+1} + \phi_h z_t + \varepsilon_{t+h}^{h}, \tag{1}$$

where the dependent variable and the lagged dependent variable are $y_{t+h}^{h} = (1200/h)$ $ln(X_{t+h}/X_t)$ and $y_t = 400ln(X_t/X_{t-1})$, respectively, X_t is the economic activity at month t, $\hat{F}_{i,t}$ is the ith principal component from a large set of predictors, z_t is either the KSR or the WXSR, and ε_{t+h}^{h} is the forecast error. The subscripts h indicate that the parameters are forecast horizon specific.

Forecasting performance is evaluated in a pseudo-out-of-sample forecasting exercise. Forecasts for industrial production, real personal consumption, nonfarm payroll employment, consumer price (CPI) inflation and personal consumption expenditures (PCE) inflation are generated for horizons of $h = 3$, 6, 9 and 12 months. We extract the factors and estimate the parameters of the forecasting models using a recursive scheme. At each forecast origin, m, k and p are selected by minimizing the Bayesian information criterion (BIC), with $1 \leq m \leq 2$, $1 \leq k \leq 4$, and $0 \leq p \leq 6$.

We quantify the out-of-sample forecast performance by computing the mean squared forecast error (MSFE) of the shadow rate forecast relative to that obtained from a benchmark model. In our framework, natural benchmark models are obtained by excluding the shadow rate from the forecasting model (1). If the relative MSFE is less than one, the shadow rate model has produced more accurate forecasts than the benchmark model. This implies that the shadow rate contains marginal predictive power. The statistical significance is evaluated using the one-sided Diebold and Mariano (1995) test with the small sample modification proposed by Harvey et al. (1997).

3. Data

We consider the shadow rates discussed in Krippner (2015b; KSR) and Wu and Xia (2016; WXSR). These shadow rates are available on the Reserve Bank of New Zealand's and the Atlanta Fed's webpages, respectively. The data refer to the rates on the last day of each month. The macroeconomic data are obtained from the St. Louis Fed's FRED-MD database, which contains 134 monthly U.S. macroeconomic variables (see McCracken and Ng, 2015). The principal components estimation of the factors require a balanced panel of data, and thus, we drop series 64 (New Orders for Consumer Goods) from the original dataset. After this modification, we have a balanced panel of 133 series from 1985:M11 to 2015:M10. A complete list of the series and transformations applied to each series is reported in Appendix A.

4. Empirical results

The out-of-sample results for the 1996:M10–2008:M12 non-ZLB period and for the 2009:M1–2016:M1 ZLB period are summarized in Tables 1 and 2, respectively. These tables show the MSFE value of the model augmented with a shadow rate relative to the MSFE value of the benchmark model.

Four main conclusions emerge from Tables 1 and 2. First, the relative MSFE values are typically above one for industrial production, real personal consumption

Table 1: Out-of-sample MSFE values for the non-ZLB period

	$h = 3$	$h = 6$	$h = 9$	$h = 12$
Industrial production				
KSR	1.066	1.176	1.249	1.324
WXSR	1.076	1.153	1.174	1.197
Real personal consumption				
KSR	1.191	1.317	1.371	1.464
WXSR	1.182	1.255	1.217	1.348
Nonfarm payroll employment				
KSR	1.244	1.403	1.463	1.516
WXSR	1.162	1.282	1.317	1.325
CPI inflation				
KSR	0.985	0.950	0.898	0.882
WXSR	0.972	0.908	0.858*	0.831*
PCE inflation				
KSR	0.987	0.986	0.972	0.973
WXSR	0.978	0.977	0.962	0.957

Notes: The out-of-sample forecasting period runs from 1996:M10 to 2008:M12. Each row reports the ratio of the MSFE of a forecasting model augmented with a shadow rate relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table 2: Out-of-sample MSFE values for the ZLB period

	$h = 3$	$h = 6$	$h = 9$	$h = 12$
Industrial production				
KSR	1.030	1.030	0.994	0.966
WXSR	1.040	1.039	0.979*	0.941
Real personal consumption				
KSR	1.208	1.303	1.181	1.273
WXSR	1.134	1.127	1.077	1.124
Nonfarm payroll employment				
KSR	1.045	1.018	0.980	0.931
WXSR	1.060	1.033	0.979	0.973
CPI inflation				
KSR	0.951	0.932	0.799	0.916
WXSR	0.826***	0.797	0.611*	0.518*
PCE inflation				
KSR	0.952	0.911	0.843	0.739
WXSR	0.827**	0.827	0.699*	0.571*

Notes: The out-of-sample forecasting period runs from 2009:M1 to 2016:M1. Each row reports the ratio of the MSFE of a forecasting model augmented with a shadow rate relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table 3: Out-of-sample performance of the WXSR versus the KSR for inflation

	$h = 3$	$h = 6$	$h = 9$	$h = 12$
CPI inflation				
Non-ZLB	0.994	0.959**	0.958	0.941
ZLB	0.858***	0.812***	0.772**	0.587***
PCE inflation				
Non-ZLB	0.967***	0.968**	0.985	0.941**
ZLB	0.861***	0.861**	0.837**	0.757**

Notes: Each row reports the ratio of the MSFE of a forecasting model augmented with the WXSR relative to the MSFE of a forecasting model augmented with the KSR. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively. The forecasting periods are as defined in Tables 1 and 2, respectively.

and nonfarm payroll employment, suggesting that the shadow rates do not contain predictive power for U.S. real activity in a data-rich environment. Second, the models augmented with the shadow rates produce more accurate inflation forecasts than the benchmark irrespective of which forecasting period or forecast horizon is considered. The improvements in forecast accuracy are often large. Therefore, the shadow rates have predictive power for inflation when the predictive information encoded in a large number of macroeconomic variables is already taken into account. This finding is important because the results in the previous literature suggest that it is difficult to predict inflation in the post-1985 period (see, e.g., Stock and Watson, 2007). Third, although the KSR is empirically more robust and more consistent with unconventional monetary policy events than the WXSR (Krippner, 2015a), the WXSR performs better in the out-of-sample forecasting exercise. Fourth, the predictive power of the shadow rates is similar in both out-of-sample periods.[1]

We formally compare the relative forecasting performance of the shadow rates for inflation in Table 3. This table reports the MSFE of the model augmented with the WXSR relative to the MSFE of the model augmented with the KSR. The relative MSFE values in Table 3 are below one. Thus, the results indicate that the WXSR is a better leading indicator than the KSR.[2]

5. Conclusions

This paper examined whether the shadow rates have out-of-sample predictive power for U.S. real economic activity and inflation in a data-rich environment. We find that the shadow rates are useful leading indicators for inflation. The shadow rates contain substantial predictive power for inflation in the non-ZLB and ZLB periods irrespective

[1] The results are very similar if alternative model specifications are considered; see Appendix B.

[2] Krippner (2015a) shows that different model specifications produce different estimates of the WXSR. Therefore, we emphasize that the WXSR forecasting results are specific to the particular WXSR we have used.

of which model specification or forecast horizon is considered. We find that the shadow rate suggested by Wu and Xia (2016) produces more accurate inflation forecasts than the shadow rate suggested by Krippner (2015b). The results show that the shadow rates do not have predictive power for real activity.

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Appendix A

Table A1: Data description

id	Mnemonic	Trans. code	Description
1	RPI	5	Real Personal Income
2	W875RX1	5	Real Personal Income ex transfer receipts
3	DPCERA3M086SBEA	5	Real Personal Consumption Expenditures
4	CMRMTSPLx	5	Real Manu. and Trade Industries Sales
5	RETAILx	5	Retail and Food Services Sales
6	INDPRO	5	IP Index
7	IPFPNSS	5	IP: Final Products and Nonindustrial Supplies
8	IPFINAL	5	IP: Final Products (Market Group)
9	IPCONGD	5	IP: Consumer Goods
10	IPDCONGD	5	IP: Durable Consumer Goods
11	IPNCONGD	5	IP: Nondurable Consumer Goods
12	IPBUSEQ	5	IP: Business Equipment
13	IPMAT	5	IP: Materials
14	IPDMAT	5	IP: Durable Materials
15	IPNMAT	5	IP: Nondurable Materials
16	IPMANSICS	5	IP: Manufacturing (SIC)
17	IPB51222s	5	IP: Residential Utilities
18	IPFUELS	5	IP: Fuels
19	NAPMPI	1	ISM Manufacturing: Production Index
20	CUMFNS	2	Capacity Utilization: Manufacturing
21	HWI	2	Help-Wanted Index for United States
22	HWIURATIO	2	Ratio of Help Wanted/No. Unemployed
23	CLF16OV	5	Civilian Labor Force
24	CE16OV	5	Civilian Employment
25	UNRATE	2	Civilian Unemployment Rate
26	UEMPMEAN	2	Average Duration of Unemployment (Weeks)
27	UEMPLT5	5	Civilians Unemployed - Less Than 5 Weeks
28	UEMP5TO14	5	Civilians Unemployed for 5-14 Weeks
29	UEMP15OV	5	Civilians Unemployed - 15 Weeks & Over
30	UEMP15T26	5	Civilians Unemployed for 15-26 Weeks
31	UEMP27OV	5	Civilians Unemployed for 27 Weeks and Over
32	CLAIMSx	5	Initial Claims
33	PAYEMS	5	All Employees: Total nonfarm
34	USGOOD	5	All Employees: Goods-Producing Industries
35	CES1021000001	5	All Employees: Mining and Logging: Mining
36	USCONS	5	All Employees: Construction
37	MANEMP	5	All Employees: Manufacturing
38	DMANEMP	5	All Employees: Durable Goods
39	NDMANEMP	5	All Employees: Nondurable Goods
40	SRVPRD	5	All Employees: Service-Providing Industries
41	USTPU	5	All Employees: Trade, Transportation & Utilities
42	USWTRADE	5	All Employees: Wholesale Trade
43	USTRADE	5	All Employees: Retail Trade
44	USFIRE	5	All Employees: Financial Activities
45	USGOVT	5	All Employees: Government
46	CES0600000007	1	Avg Weekly Hours: Goods-Producing
47	AWOTMAN	2	Avg Weekly Overtime Hours: Manufacturing
48	AWHMAN	1	Avg Weekly Hours: Manufacturing
49	NAPMEI	1	ISM Manufacturing: Employment Index
50	HOUST	4	Housing Starts: Total New Privately Owned
51	HOUSTNE	4	Housing Starts, Northeast
52	HOUSTMW	4	Housing Starts, Midwest
53	HOUSTS	4	Housing Starts, South
54	HOUSTW	4	Housing Starts, West
55	PERMIT	4	New Private Housing Permits (SAAR)
56	PERMITNE	4	New Private Housing Premits, Northeast (SAAR)
57	PERMITMW	4	New Private Housing Permits, Midwest (SAAR)
58	PERMITS	4	New Private Housing Permits, South (SAAR)

(Continued)

Table A1 – *(Continued)*

id	Mnemonic	Trans. code	Description
59	PERMITW	4	New Private Housing Permits, West (SAAR)
60	NAPM	1	ISM: PMI Composite Index
61	NAPMNOI	1	ISM: New Orders Index
62	NAPMSDI	1	ISM: Supplier Deliveries Index
63	NAPMII	1	ISM: Inventories Index
65	AMDMNOx	5	New Orders for Durable Goods
66	ANDENOx	5	New Orders for Nondefense Capital Goods
67	AMDMUOx	5	Unfilled Orders for Durable Goods
68	BUSINVx	5	Total Business Inventories
69	ISRATIOx	2	Total Business: Inventories to Sales Ratio
70	M1SL	6	M1 Money Stock
71	M2SL	6	M2 Money Stock
72	M2REAL	5	Real M2 Money Stock
73	AMBSL	6	St. Louis Adjusted Monetary Base
74	TOTRESNS	6	Total Reserves of Depository Institutions
75	NONBORRES	7	Reserves of Depository Institutions, Nonborrowed
76	BUSLOANS	6	Commercial and Industrial Loans, All Commercial Banks
77	REALLN	6	Real Estate Loans at All Commercial Banks
78	NONREVSL	6	Total Nonrevolving Credit Owner and Securitized Outstanding
79	CONSPI	2	Nonrevolving Consumer Credit to Personal Income
80	S & P 500	5	S&P's Common Stock Price Index: Composite
81	S & P: indust	5	S&P's Common Stock Price Index: Industrials
82	S & P div yield	2	S&P's Composite Common Stock: Dividend Yield
83	S & P PE ratio	5	S&P's Composite Common Stock: Price-Earnings Ratio
84	FEDFUNDS	2	Effective Federal Funds Rate
85	CP3Mx	2	3-Month AA Financial Commercial Paper Rate
86	TB3MS	2	3-Month Treasury Bill
87	TB6MS	2	6-Month Treasury Bill
88	GS1	2	1-Year Treasury Rate
89	GS5	2	5-Year Treasury Rate
90	GS10	2	10-Year Treasury Rate
91	AAA	2	Moody's Seasoned Aaa Corporate Bond Yield
92	BAA	2	Moody's Seasoned Baa Corporate Bond Yield
93	COMPAPFFx	1	3-Month Commercial Paper Minus FEDFUNDS
94	TB3SMFFM	1	3-Month Treasury C Minus FEDFUNDS
95	TB6SMFFM	1	6-Month Treasury C Minus FEDFUNDS
96	T1YFFM	1	1-Year Treasury C Minus FEDFUNDS
97	T5YFFM	1	5-Year Treasury C Minus FEDFUNDS
98	T10YFFM	1	10-Year Treasury C Minus FEDFUNDS
99	AAAFFM	1	Moody's Aaa Corporate Bond Minus FEDFUNDS
100	BAAFFM	1	Moody's Baa Corporate Bond Minus FEDFUNDS
101	TWEXMMTH	5	Trade Weighted U.S. Dollar Index: Major Currencies
102	EXSZUSx	5	Switzerland / U.S. Foreign Exchange Rate
103	EXJPUSx	5	Japan / U.S. Foreign Exchange Rate
104	EXUSUKx	5	U.S. / U.K. Foreign Exchange Rate
105	EXCAUSx	5	Canada / U.S. Foreign Exchange Rate
106	PPIFGS	6	PPI: Finished Goods
107	PPIFCG	6	PPI: Finished Consumer Goods
108	PPIITM	6	PPI: Intermediate Materials
109	PPICRM	6	PPI: Crude Materials
110	OILPRICEx	6	Crude Oil, spliced WTI and Cushing
111	PPICMM	6	PPI: Metals and Metal Products
112	NAPMPRI	1	ISM Manufacturing: Prices Index
113	CPIAUCSL	6	CPI: All Items
114	CPIAPPSL	6	CPI: Apparel
115	CPITRNSL	6	CPI: Transportation
116	CPIMEDSL	6	CPI: Medical Care
117	CUSR0000SAC	6	CPI: Commodities
118	CUUR0000SAD	6	CPI: Durables
119	CUSR0000SAS	6	CPI: Services
120	CPIULFSL	6	CPI: All Items Less Food
121	CUUR0000SA0L2	6	CPI: All Items Less Shelter
122	CUSR0000SA0L5	6	CPI: All Items Less Medical Care
123	PCEPI	6	Personal Cons. Expend.: Chain Price Index
124	DDURRG3M086SBEA	6	Personal Cons. Expend.: Durable Goods

(Continued)

Table A1 – *(Continued)*

id	Mnemonic	Trans. code	Description
125	DNDGRG3M086SBEA	6	Personal Cons. Expend.: Nondurable Goods
126	DSERRG3M086SBEA	6	Personal Cons. Expend.: Services
127	CES0600000008	6	Avg Hourly Earnings: Goods-Producing
128	CES2000000008	6	Avg Hourly Earnings: Construction
129	CES3000000008	6	Avg Hourly Earnings: Manufacturing
130	UMCSENTx	2	Consumer Sentiment Index
131	MZMSL	6	MZM Money Stock
132	DTCOLNVHFNM	6	Consumer Motor Vehicle Loans Outstanding
133	DTCTHFNM	6	Total Consumer Loans and Leases Outstanding
134	INVEST	6	Securities in Bank Credit at All Commercial Banks

Notes: The transformation code (column 3) denotes the transformation applied to the variable before the principal components are calculated. The transformation codes are 1 = no transformation, 2 = first difference, 3 = second difference, 4 = natural logarithm, 5 = first difference of logarithms and 6 = second difference of logarithms. The data sample is 1985:M11–2015:M10. The data source is the FRED-MD database.

Appendix B

In this Appendix, we present the results for several variants of the forecasting model (1). The first, denoted by DIAR, includes a contemporaneous shadow rate and lags of $\hat{F}_{i,t}$ and y_t, with m, k and p selected by minimizing the Bayesian information criterion (BIC), with $1 \leq m \leq 2$, $1 \leq k \leq 4$, and $0 \leq p \leq 6$. The second variant, denoted by K1–K4, includes a fixed number of factors ($k = 1, \ldots, 4$) and a contemporaneous value of the shadow rate. The third variant includes the contemporaneous value of the shadow rate and lags of y_t. The number of autoregressive lags is selected by the BIC, with $0 \leq p \leq 6$. This variant is denoted by AR in the following tables.

The results of this sensitivity analysis, reported in Tables B1–B3, corroborate the findings in Tables 1–3.

Table B1: Out-of-sample MSFE values for the non-ZLB period

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
A) *Industrial production*					
KSR	DIAR	1.066	1.176	1.249	1.324
	K1	1.066	1.116	1.149	1.183
	K2	1.084	1.160	1.210	1.248
	K3	1.085	1.159	1.208	1.249
	K4	1.075	1.158	1.224	1.290
	AR	1.061	1.099	1.139	1.176
WXSR	DIAR	1.076	1.153	1.174	1.197
	K1	1.065	1.094	1.102	1.112
	K2	1.100	1.154	1.174	1.183
	K3	1.097	1.147	1.166	1.179
	K4	1.084	1.148	1.186	1.227
	AR	1.074	1.092	1.101	1.108
B) *Real personal consumption*					
KSR	DIAR	1.191	1.317	1.371	1.464
	K1	1.077	1.120	1.152	1.175
	K2	1.135	1.193	1.203	1.212
	K3	1.150	1.220	1.234	1.241
	K4	1.180	1.281	1.324	1.337
	AR	1.075	1.133	1.138	1.174

(Continued)

Table B1 – *(Continued)*

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
WXSR	DIAR	1.182	1.255	1.217	1.348
	K1	1.077	1.093	1.092	1.118
	K2	1.159	1.166	1.126	1.148
	K3	1.183	1.197	1.153	1.172
	K4	1.221	1.270	1.258	1.291
	AR	1.061	1.089	1.077	1.108
C) *Nonfarm payroll employment*					
KSR	DIAR	1.244	1.403	1.463	1.516
	K1	0.846***	0.999	1.109	1.200
	K2	1.017	1.210	1.321	1.405
	K3	1.052	1.249	1.346	1.422
	K4	1.174	1.372	1.457	1.522
	AR	1.103	1.166	1.194	1.225
WXSR	DIAR	1.162	1.282	1.317	1.325
	K1	0.762***	0.879**	0.963	1.034
	K2	0.958	1.109	1.181	1.238
	K3	1.001	1.152	1.206	1.251
	K4	1.106	1.257	1.300	1.337
	AR	1.070	1.097	1.102	1.116
D) *CPI inflation*					
KSR	DIAR	0.985	0.950	0.898	0.882
	K1	0.982	0.946	0.904	0.920
	K2	0.986	0.933	0.866	0.856
	K3	1.030	0.981	0.902	0.883
	K4	1.052	1.057	0.950	0.882
	AR	0.971	0.932	0.888	0.908
WXSR	DIAR	0.972	0.908	0.858*	0.831*
	K1	0.964	0.908	0.864	0.858
	K2	0.965	0.891*	0.823*	0.798*
	K3	1.015	0.949	0.871	0.835
	K4	1.034	1.021	0.915	0.824
	AR	0.968*	0.920*	0.882*	0.888
E) *PCE inflation*					
KSR	DIAR	0.987	0.986	0.972	0.973
	K1	0.997	0.991	0.989	1.035
	K2	0.988	0.962	0.933	0.954
	K3	1.044	1.016	0.976	0.989
	K4	1.072	1.084	1.005	0.969
	AR	0.984	0.994	0.974	1.001

(Continued)

Table B1 – *(Continued)*

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
WXSR	DIAR	0.978	0.977	0.962	0.957
	K1	0.971	0.943	0.936	0.959
	K2	0.959	0.908	0.874	0.878
	K3	1.028	0.977	0.931	0.925
	K4	1.060	1.051	0.961	0.895
	AR	0.980	0.986	0.970	0.980

Notes: The out-of-sample forecasting period runs from 1996:M10 to 2008:M12. KSR denotes the Krippner (2015b) shadow rate, and WXSR denotes the Wu and Xia (2016) shadow rate. Each row reports the ratio of the MSFE of a forecasting model augmented with a shadow rate relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table B2: Out-of-sample MSFE values for the ZLB period

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
A) *Industrial production*					
KSR	DIAR	1.030	1.030	0.994	0.966
	K1	1.025	1.058	1.071	1.058
	K2	0.997	1.026	1.039	1.015
	K3	1.012	1.009	1.005	0.995
	K4	1.006	1.005	1.005	0.995
	AR	1.027	1.055	1.073	1.039
WXSR	DIAR	1.040	1.039	0.979*	0.941
	K1	1.024	1.070	1.094	1.076
	K2	0.984	1.022	1.045	1.015
	K3	0.993	0.983	0.976	0.953
	K4	0.986**	0.979*	0.973	0.949
	AR	1.020	1.048	1.072	1.035
B) *Real personal consumption*					
KSR	DIAR	1.208	1.303	1.181	1.273
	K1	0.943*	0.963	1.123	1.317
	K2	1.150	1.314	1.436	1.469
	K3	1.127	1.260	1.418	1.472
	K4	1.127	1.273	1.480	1.519
	AR	0.913**	1.047	1.156	1.297
WXSR	DIAR	1.134	1.127	1.077	1.124
	K1	0.973	0.960	1.018	1.136
	K2	1.115	1.175	1.177	1.199
	K3	1.102	1.165	1.173	1.196
	K4	1.115	1.177	1.219	1.257
	AR	0.939*	1.049	1.101	1.207
C) *Nonfarm payroll employment*					
KSR	DIAR	1.045	1.018	0.980	0.931
	K1	1.677	1.681	1.600	1.434
	K2	1.279	1.307	1.342	1.316
	K3	1.261	1.213	1.238	1.316
	K4	1.287	1.246	1.269	1.315
	AR	1.020	0.984	0.954	0.937
WXSR	DIAR	1.060	1.033	0.979	0.973
	K1	1.717	1.809	1.796	1.636
	K2	1.321	1.430	1.528	1.513
	K3	1.266	1.233	1.279	1.382
	K4	1.262	1.245	1.308	1.406

(Continued)

Table B2 – *(Continued)*

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
	AR	1.007	0.971	0.940*	0.901**
D) *CPI inflation*					
KSR	DIAR	0.951	0.932	0.799	0.916
	K1	0.902	0.855	0.796	0.683
	K2	0.993	1.018	0.988	0.934
	K3	0.918*	0.915	0.882	0.838
	K4	0.871**	0.860	0.842	0.864
	AR	0.924	0.865	0.788	0.716
WXSR	DIAR	0.826***	0.797	0.611*	0.518*
	K1	0.813**	0.717*	0.611*	0.477*
	K2	0.897**	0.856	0.748	0.644
	K3	0.793***	0.727**	0.611**	0.507*
	K4	0.763***	0.696***	0.596**	0.531*
	AR	0.793**	0.641*	0.515*	0.423*
E) *PCE inflation*					
KSR	DIAR	0.952	0.911	0.843	0.739
	K1	0.913	0.882	0.831	0.724
	K2	0.994	1.023	1.012	0.959
	K3	0.904**	0.903	0.873	0.831
	K4	0.867***	0.864*	0.868	0.914
	AR	0.934	0.889	0.843	0.718
WXSR	DIAR	0.827**	0.827	0.699*	0.571*
	K1	0.832**	0.749	0.654	0.525
	K2	0.899*	0.859	0.771	0.674
	K3	0.766***	0.694**	0.580**	0.487*
	K4	0.742***	0.674***	0.589**	0.555
	AR	0.817**	0.683*	0.612*	0.485*

Notes: The out-of-sample forecasting period runs from 2009:M1 to 2016:M1. KSR denotes the Krippner (2015b) shadow rate, and WXSR denotes the Wu and Xia (2016) shadow rate. Each row reports the ratio of the MSFE of a forecasting model augmented with a shadow rate relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table B3: Out-of-sample performance of the WXSR versus the KSR for inflation

	Model	$h = 3$	$h = 6$	$h = 9$	$h = 12$
CPI inflation					
Non-ZLB	DIAR	0.994	0.959**	0.958	0.941
	K1	0.982*	0.959**	0.956	0.933
	K2	0.979**	0.955**	0.951*	0.932*
	K3	0.985*	0.967*	0.965	0.945
	K4	0.982*	0.966*	0.963	0.934*
	AR	0.997	0.987	1.002	0.978
ZLB	DIAR	0.858***	0.812***	0.772**	0.587***
	K1	0.902***	0.839**	0.768**	0.699**
	K2	0.903***	0.841**	0.756**	0.690**
	K3	0.864***	0.795***	0.693***	0.605**
	K4	0.876***	0.810***	0.707**	0.614**
	AR	0.855***	0.741***	0.653***	0.593***
PCE inflation					
Non-ZLB	DIAR	0.967***	0.968**	0.985	0.941**
	K1	0.974**	0.951**	0.946*	0.926*
	K2	0.971**	0.944***	0.936**	0.920**
	K3	0.985*	0.962**	0.954*	0.935*
	K4	0.989	0.970*	0.956*	0.924*
	AR	1.001	0.978*	0.990	0.975
ZLB	DIAR	0.861***	0.861**	0.837**	0.757**
	K1	0.911***	0.849**	0.786**	0.726**
	K2	0.904***	0.839**	0.762**	0.703**
	K3	0.847***	0.769***	0.664***	0.586***
	K4	0.856***	0.781***	0.679***	0.607***
	AR	0.875***	0.768***	0.726***	0.716**

Notes: KSR denotes the Krippner (2015b) shadow rate, and WXSR denotes the Wu and Xia (2016) shadow rate. Each row reports the ratio of the MSFE of a forecasting model augmented with the WXSR relative to the MSFE of a forecasting model augmented with the KSR. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test at the 1% (***), 5% (**), and 10% (*) significance levels, respectively. The forecasting periods are as defined in Tables 1 and 2, respectively.